NAG Toolbox for MATLAB

f08nf

1 Purpose

f08nf generates the real orthogonal matrix Q which was determined by f08ne when reducing a real general matrix A to Hessenberg form.

2 Syntax

```
[a, info] = f08nf(ilo, ihi, a, tau, 'n', n)
```

3 Description

f08nf is intended to be used following a call to f08ne, which reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation: $A = QHQ^{\mathrm{T}}$. f08ne represents the matrix Q as a product of $i_{\mathrm{hi}} - i_{\mathrm{lo}}$ elementary reflectors. Here i_{lo} and i_{hi} are values determined by f08nh when balancing the matrix; if the matrix has not been balanced, $i_{\mathrm{lo}} = 1$ and $i_{\mathrm{hi}} = n$.

This function may be used to generate Q explicitly as a square matrix. Q has the structure:

$$Q = \begin{pmatrix} I & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & I \end{pmatrix}$$

where Q_{22} occupies rows and columns i_{lo} to i_{hi}.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **ilo – int32 scalar**

2: **ihi – int32 scalar**

These **must** be the same parameters **ilo** and **ihi**, respectively, as supplied to f08ne.

Constraints:

if $\mathbf{n} > 0$, $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq \mathbf{n}$;
if $\mathbf{n} = 0$, $\mathbf{ilo} = 1$ and $\mathbf{ihi} = 0$.

3: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the vectors which define the elementary reflectors, as returned by f08ne.

4: **tau(∗) – double array**

Note: the dimension of the array **tau** must be at least $\max(1, \mathbf{n} - 1)$.

Further details of the elementary reflectors, as returned by f08ne.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix Q.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n orthogonal matrix Q.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **n**, 2: **ilo**, 3: **ihi**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}q^3$, where $q = i_{\mathrm{hi}} - i_{\mathrm{lo}}$.

The complex analogue of this function is f08nt.

9 Example

```
ilo = int32(1);
ihi = int32(4);
a = [0.35, -0.1159524296205035, -0.3886010343233214, -0.2941840753473021;
          -0.5140038910358558,  0.1224867524602574,  0.1003597896821503,
0.1125618799705319;
```

```
        -0.7284721282927631,  0.6442636185270625,  -0.1357001717571131,  -
0.09768162270493326;
          0.4139046183481608,  -0.1665445794905699,  0.4262443722078449,
0.1632134192968561];
tau = [1.175095950769217;
     1.946022972139863;
     0];
[aOut, info] = f08nf(ilo, ihi, a, tau)
```

```
aOut =
    1.0000         0         0         0
         0   -0.1751   -0.9675   -0.1827
         0    0.8560   -0.2413    0.4572
         0   -0.4864   -0.0763    0.8704
info =
           0
```